INVESTIGATION ON MISSING MASS SPECTRUM OF D (K^-, n) Λ (1405) REACTION

Win Win Maw[1], Thida Wint[2], Khin Swe Myint[3], Yoshinori Akaishi[4], Toshimitsu Yamazaki[5]

Abstract

The aim of this research work is to investigate the missing mass spectrum of D (K^-, n) Λ (1405) reaction process of J-PARC (Japan Proton Accelerator Research Complex) E31 experiment. This experiment which was conducted at J-PARC with 1.0 GeV/c incident momentum of K^- on target deuterium. This reaction is expected to enhance a virtual $\bar{K}N$ scattering process, where a K^- beam kicks a neutron out of the deuteron target in a forward angle and is slowing down to form a Λ(1405) with a residual nucleon. We have calculated the missing mass spectrum of D (K^-, n) Y reaction with Green's function method by using YA (Yamazaki and Akaishi) potential for $\bar{K}N$ interaction. It is observed that the missing mass spectrum of the D (K^-, n) Y reaction at a neutron forward angle has two peaks, one below the K^-p threshold and another above the threshold. The former peak represents Λ (1405) quasi-bound state while the latter is a quasi-free K^-p peak.

Key words: virtual $\bar{K}N$ scattering, missing mass, threshold, quasi-free, quasi-bound.

Introduction

An antikaon ($\bar{K}$) and a nucleus may form a bound state (a kaonic nucleus), due to the strong attraction between $\bar{K}$ and nucleon in an isospin, I = 0 state. Λ (1405) resonance state is nominally accepted as a bound state of K^-p system which lies in the continuum region of π Σ, having strangeness, S = -1, total charge, Q = 0, isospin, I = 0 and spin parity, $J^p = 1/2^-$.

The PDG value of the mass and width of this Λ (1405) resonance state or often known as Λ* is $1405.1^{+1.3}_{-1.0}$ MeV/c^2 and 50.5 ± 2.0 MeV (Particle Data Group K. A. Olive *et al.*, (2014)) with 27 MeV binding energy with respect to $\bar{K}N$ threshold.

However, chiral unitary model claims that Λ (1405) may have two pole structure; one is mainly coupled to πΣ state and the other is to $\bar{K}N$ state which are located at different positions, (1390-132i) MeV and (1426-32i) MeV, respectively (T. Hyodo and A. Weise, (2008)). As a consequence, the resonance position of the Λ (1405) is 1420 MeV/c^2 and the binding energy is as shallow as 15 MeV.

In K^-p reactions at 4.2 GeV/c, the mass and width of Λ (1405).resonance were obtained to be 1400.5 ± 4.0 MeV/c^2 and 50.0 ± 2.0 MeV from production of Λ (1405) (R. J. Hemingway, (1985)) by Dalitz and Deloff (R. H. Dalitz and A. Deloff, (1991)). It is interpreted as a quasi-bound state of $\bar{K}N$ coupled with continuum stat of πΣ. Esmaili *et al.*,(J. Esmaili, Y. Akaishi and T. Yamazaki, (2010), (2011)) analyzed old bubble-chamber of stopped-K^- on ^{4}He (B. Riley *et al.*,

[1] Dr, Assistant lecturer, Physics Department, Mandalay University, Mandalay, Myanmar
[2] Associate Professor, Physics Department, University of Distance Education Mandalay, Mandalay, Myanmar
[3] Emeritus Professor, Physics Department, Mandalay University, Mandalay, Myanmar
[4] Dr, Professor, College of Science and Technology, Nihon University, Chiba, Japan and RIKEN Nishina Center, Saitama, Japan
[5] Professor, Department of Physics, University of Tokyo, Bunkyo-ku, Tokyo 113-0033, Japan

(1975)) with a resonance capture process, and found the best-fit value of mass and width for the Λ (1405) are $1405.1^{+1.3}_{-1.0}$ MeV/c^2 and $24.0^{+4.0}_{-3.0}$ MeV.

Maryam *et al*. (M. Hassanvand, Y. Akaishi, T. Yamazaki, (2015)) have calculated the $\Lambda(1405) \rightarrow (\pi\Sigma)^0$ invariant mass spectra produced in the reaction $K^- + p \rightarrow \Sigma^+(1660) + \pi^-$, followed by $\Sigma^+(1660) \rightarrow \Lambda(1405) + \pi^+ \rightarrow \Sigma\pi + \pi^+$, processes at p(K$^-$) = 4.2 GeV/c.

An experiment which has reported Λ (1405) resonance state in the reaction $K^- p \rightarrow \Sigma\ 3\pi$ at 1.15 GeV/c was conducted by Alston *et al*., (R.H. Alston *et al*., (1961)). The values of mass and width of this resonance state are 1405 MeV/c^2 and 20 MeV, respectively.

Another experiment in the reaction of $\pi^- p \rightarrow \Sigma\ \pi\ K$ at 1.69 GeV/c π^- beam has been reported for searching Λ (1405) resonance state by Thomas *et al.*, (D.W. Thomas *et al.*,(1977)). The mass and width of this resonance state from this experiment are ~1405 MeV/c^2 and 45 to 55 MeV, respectively.

Momentum Transfer of Emitted Neutron

First, the momentum transfer of the emitted neutron which is an important for Λ (1405) formation processes are calculated. The momentum transfer of emitted neutron (n_1) and product kaon (K_1^-) from the elementary process n (K^-, n_1) K_1^- are calculated by using Newton-Raphson method. Then, the missing mass of $K^- p$ is calculated after having obtained the momentum transfer of neutron (n_1).

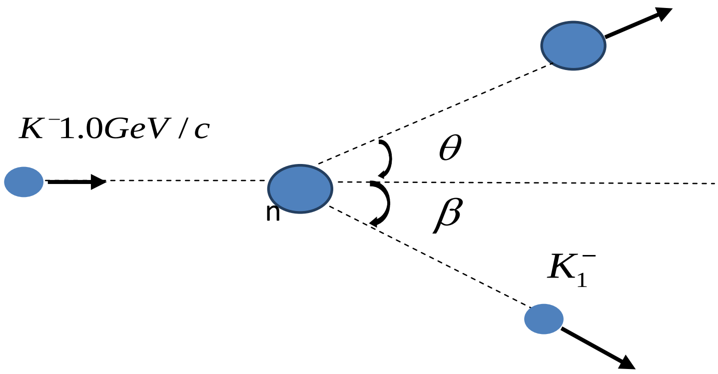

Figure 1 Schematic diagram of the elementary process

Elementary process is

$$K^- + n \rightarrow n_1 + K_1^- \quad (1)$$

By the law of conservation of energy, we have

$$E_{K^-} + E_n = E_{n_1} + E_{K_1^-}$$

where,

$$E = \sqrt{p^2c^2 + m^2c^4} \quad (2)$$

Equation (2) becomes,

$$\sqrt{p_{K^-}^2 c^2 + m_{K^-}^2 c^4} + \sqrt{p_n^2 c^2 + M_n^2 c^4} = \sqrt{p_{n_1}^2 c^2 + M_n^2 c^4} + \sqrt{p_{K_1^-}^2 c^2 + m_{K^-}^2 c^4} \tag{3}$$

By the law of momentum conservation,

$$\vec{p}_{K^-} + \vec{p}_n = \vec{p}_{n_1} + \vec{p}_{K_1^-} \tag{4}$$

$$\vec{p}_{K_1^-} = \vec{p}_{K^-} - \vec{p}_{n_1}$$

$$p_{K_1^-}^2 = p_{K^-}^2 + p_{n_1}^2 - 2p_{K^-} p_{n_1} \cos(\theta) \tag{5}$$

By substituting equation (5) in to equation (3), we obtain

$$\sqrt{p_{K^-}^2 + m_{K^-}^2} + M_n = \sqrt{p_{n_1}^2 + M_n^2} + \sqrt{(p_{K^-}^2 + p_{n_1}^2 - 2p_{K^-} p_{n_1} \cos(\theta)) + m_{K^-}^2}\ .$$

$$\sqrt{p_{n_1}^2 + M_n^2} + \sqrt{(p_{K^-}^2 + p_{n_1}^2 - 2p_{K^-} p_{n_1} \cos(\theta)) + m_{K^-}^2} - \sqrt{p_{K^-}^2 + m_{K^-}^2} - M_n = 0 \tag{6}$$

The momentum of emitted neutron (n_1) can be obtained by solving the equation (6) by using Newton-Raphson Method. This method is numerical method applied to find the root of a function f (p_{n1}) equal to zero.

Missing mass formula for D (K^- , n) Y

The missing mass formula for D (K^-, n) Y reaction is calculated. If we know the emitted neutron momentum, we can get the missing mass spectrum.

$$K^- + D \rightarrow n + Y \qquad (Y = \Lambda^* \text{ or } K^- + p) \tag{7}$$

$$E_Y^2 = P_Y^2 c^2 + M_Y^2 c^4 \tag{8}$$

$$M_Y^2 c^4 = E_Y^2 - P_Y^2 c^2 \tag{9}$$

For a system Y, missing mass is defined as

$$M_Y c^2 = \sqrt{E_Y^2 - P_Y^2 c^2} \tag{10}$$

By the law of conservation of energy,

Let $E_{init} = E_{K^-} + E_D$ and $E_n = E_{obs}$ (11)

$$E_{init} = E_{obs} + E_Y \tag{12}$$

$$E_Y = E_{init} - E_{obs} \tag{13}$$

By the law of conservation of momentum,

$$\vec{P}_{K^-} + \vec{P}_D = \vec{P}_n + \vec{P}_Y \tag{14}$$

Let $\vec{p}_{init} = \vec{p}_{K^-} + \vec{p}_D$ and $\vec{p}_n = \vec{p}_{obs}$

$$\vec{p}_{init} = \vec{p}_{obs} + \vec{p}_Y \tag{15}$$

$$\vec{p}_Y = \vec{p}_{init} - \vec{p}_{obs} \tag{16}$$

By substituting equation (13) and equation (16) into equation (10), the equation (10) becomes

$$M_Y c^2 = \sqrt{(E_{init} - E_{obs})^2 - (p^2_{init} + p^2_{obs} - 2P_{init} p_{obs} \cos(\theta))c^2} \tag{17}$$

We can get the missing mass spectrum by using the above formula.

Differential cross section for D (K$^-$, n) Λ (1405)

The differential cross section is the most important factor to determine the probability of the reaction. Therefore, we are going to determine the differential cross section and spectral function for D (K$^-$, n) Λ (1405) reaction. The differential cross section is defined as the transition rate per incident flux. According to the Fermi's Golden Rule, the transition rate W_{fi},

$$W_{fi} = \frac{2\pi}{\hbar} |T_{fi}|^2 \delta\left(E_i - E_f^{(n)}\right) \rho(E). \tag{18}$$

where, T_{fi} is the transition matrix element, $\delta\left(E_i - E_f^{(n)}\right)$ is the energy conservation term and $\rho(E)$ is the density of allowed states.

The expression for differential cross section with neutron momentum between $\vec{k}_n$ and $\vec{k}_n + d\vec{k}_n$ and with Λ (1405) momentum between $\vec{K}$ and $\vec{K} + d\vec{K}$ is written as follows:

$$d^6\sigma = \frac{L^3}{v_0} \frac{2\pi}{\hbar} \sum_n \delta(E_i - E_f^{(n)}) (\frac{L}{2\pi})^3 d\vec{k}_n (\frac{L}{2\pi})^3 d\vec{K} \left|T_{fi}^{(n)}\right|^2 . \tag{19}$$

where, $\frac{v_0}{L^3}$ = incident flux, incident kaon velocity, $v_0 = \frac{\hbar k_0 c^2}{E_0}$ and $\left(\frac{L}{2\pi}\right)^3 d\vec{k}_n \left(\frac{L}{2\pi}\right)^3 d\vec{K}$ = phase space.

The differential cross section contains per incident flux, energy conservation term, phase space and transition matrix element. First, we will calculate the transition matrix element.

Transition matrix element

We considered the elementary process of reaction $K^- + D \rightarrow n + \Lambda(1405)$

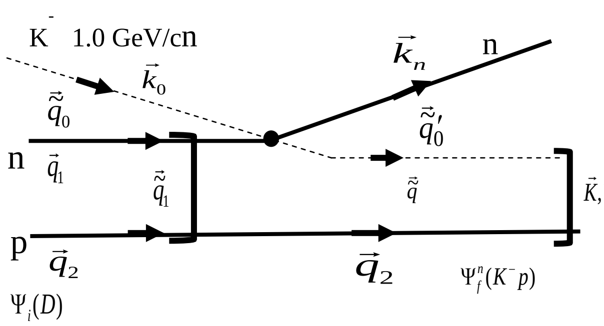

Figure 2 Schematic diagram of the reaction D (K$^-$, n) Λ (1405)

Transition matrix for D (K^- , n) Λ (1405) reaction can be described as follows:

$$T_{fi}^{(n)} = \langle \text{final state} | T | \text{initial state} \rangle . \tag{20}$$

$$T_{fi}^{(n)} = \left\langle \Psi_f^n (K^-p), \vec{K}, \vec{k}_n \; | T | \Psi_i (D), \vec{0}, \vec{k}_0 \right\rangle \tag{21}$$

where, T = transition operator, $\Psi_f^n(K^-p)$ involves $\vec{q}_2$ and $\vec{q}_0'$, $\Psi_i(D)$ involves $\vec{q}_1$ and $\vec{q}_2$.

Relation between $\langle \vec{r} | \vec{k} \rangle$ and $\langle \vec{r} | \vec{k}]$ is $\langle \vec{r} | \vec{k} \rangle = \left(\frac{L}{2\pi} \right)^{\frac{3}{2}} \langle \vec{r} | \vec{k}]$.

Equation (21) is rewritten in terms of internal coordinates with the aid completeness relation.

$$T_{fi}^{(n)} = \left(\frac{L}{2\pi} \right)^9 \iiint d\vec{q}_1 d\vec{q}_2 \, d\vec{q}_0' \left\langle \Psi_f^n (K^-p), \vec{K} \middle| \vec{q}_0', \vec{q}_2 \right] \left[\vec{q}_0', \vec{k}_n \middle| T_{K^-n} \middle| \vec{k}_0, \vec{q}_1 \right] \left[\vec{q}_1, \vec{q}_2 \middle| \vec{0}, \Psi_i (D) \right\rangle \tag{22}$$

where,
$$\int d\vec{q} |\vec{q}\rangle\langle\vec{q}| = \int d\vec{q} \left(\frac{L}{2\pi} \right)^3 |\vec{q}][\vec{q}| = 1 .$$

By solving equation (21), he final expression of transition matrix element as follows.

$$T_{fi}^{(n)} = \int d\vec{q}_1 \left\langle \Psi_f^{(n)} (K^-p) \middle| \vec{\tilde{q}} \right] \delta(\vec{K} + \vec{k}_n - \vec{k}_0) \; [\vec{\tilde{q}}_0' | t_{K^-n} | \vec{\tilde{q}}_0][\vec{\tilde{q}}_1 | \Psi_i (D) \rangle . \tag{23}$$

After calculating the transition matrix element, we will be calculated the differential cross section for this reaction. After that, by substituting the transition matrix element into equation (19), the differential cross section for this reaction becomes,

$$d^6\sigma = \frac{L^3}{v_0} \frac{2\pi}{\hbar} \sum_n \delta(E_i - E_f^{(n)}) (\frac{L}{2\pi})^3 d\vec{k}_n (\frac{L}{2\pi})^3 d\vec{K} \delta^2 (\vec{K} + \vec{k}_n - \vec{k}_0)$$

$$\times \left| \int d\vec{q}_1 \left\langle \Psi_f^{(n)})(K^-p) \middle| \vec{\tilde{q}} \right] \left[\vec{\tilde{q}}_0' \middle| t_{K^-n} \middle| \vec{\tilde{q}}_0 \right] \left[\vec{q}_1 \middle| \Psi_i (D) \right\rangle \right|^2 . \tag{24}$$

By integrating the above equation (24) with $d\vec{K}$, .

$$d^3\sigma = \frac{L^3}{v_0} \frac{2\pi}{\hbar} \sum_n \delta(E_i - E_f^n) (\frac{L}{2\pi})^9 d\vec{k}_n \left| \int d\vec{q}_1 \left\langle \Psi_f^n (K^-p) \middle| \vec{\tilde{q}} \right] \left[\vec{\tilde{q}}_0' \middle| t_{K^-n} \middle| \vec{\tilde{q}}_0 \right] \left[\vec{q}_1 \middle| \Psi_i (D) \right\rangle \right|^2 . \tag{25}$$

$v_0 = \dfrac{\hbar k_0 c^2}{E_0}$ is substituted into equation (25). The equation (27) becomes

$$d^3\sigma = \frac{L^3 E_0}{\hbar k_0 c^2} \frac{2\pi}{\hbar} \sum_n \delta(E_i - E_f^{(n)}) (\frac{L}{2\pi})^9 d\vec{k}_n$$

$$\times \left| \int d\vec{q}_1 \left\langle \Psi_f^{(n)} (K^-p) \middle| \vec{\tilde{q}} \right] \left[\vec{\tilde{q}}_0' \middle| t_{K^-n} \middle| \vec{\tilde{q}}_0 \right] \left[\vec{q}_1 \middle| \Psi_i (D) \right\rangle \right|^2 . \tag{26}$$

Equation (26) can be expressed in terms of delta function normalization. The equation (26) becomes

$$d^3\sigma = \frac{(2\pi)^4}{\hbar^2 k_0 c^2} E_0 \sum_n \delta(E_i - E_f^{(n)}) d\vec{k}_n \left|\left\langle t_{K^-n}\right\rangle\right|^2 \left|\int d\vec{q}_1 \left\langle \Psi_f^{(n)}(K^-p) \middle| \vec{\tilde{q}} \right\rangle \left\langle \vec{q}_1 \middle| \Psi_i(D) \right\rangle\right|^2 \tag{27}$$

$$\because \left\langle t_{K^-n} \right\rangle = \left\langle \vec{\tilde{q}}' \middle| t_{K^-n} \middle| \vec{\tilde{q}}_0 \right\rangle .$$

Calculation of Energy Conservation Term

Energy of initial state, E_i, $E_i = E_0 + M_n c^2 + M_p c^2 - BE$

(28)

where, BE is the binding energy of deuteron.

Energy of final state, E_f

$$E_f^{(n)} = E_n + (m_{K^-} c^2 + M_p c^2) + \left\langle \Psi_f^{(n)} \middle| H_{K^-p} \middle| \Psi_f^{(n)} \right\rangle + \frac{\hbar^2 \vec{K}^2}{2(m_{K^-} + M_p)} \tag{29}$$

$$E = E_0 + M_n c^2 + M_p c^2 - BE - E_n - (m_{K^-} c^2 + M_p c^2) - \frac{\hbar^2 (\vec{k}_0 - \vec{k}_n)^2}{2(m_{K^-} + M_p)} \tag{30}$$

where, $\frac{\hbar^2 (\vec{k}_0 - \vec{k}_n)^2}{2(m_{K^-} + M_p)}$ is the recoil energy and $\left\langle \Psi_f^{(n)} \middle| H_{K^-p} \middle| \Psi_f^{(n)} \right\rangle$ is the excitation energy of final state.

$$\delta(E_i - E_f^{(n)}) = \left\langle \Psi_f^{(n)} \middle| \delta\left(E - H_{K^-p}\right) \middle| \Psi_f^{(n)} \right\rangle \tag{31}$$

To obtain the $\delta(E_i - E_f^{(n)})$, the Cauchy's principle is used.

$$\frac{1}{x + i\varepsilon} = \frac{p}{x} - i\pi\delta(x)$$

where, P is the principle value.

$$\delta(x) = -\frac{1}{\pi} \mathrm{Im} \frac{1}{x + i\varepsilon}$$

And then, we can expressed the energy conservation term as follows:

$$\delta(E_i - E_f^{(n)}) = -\frac{1}{\pi} \mathrm{Im} \frac{1}{E - H_{K^-p} + i\varepsilon} \tag{32}$$

The equation (27) becomes

$$d^3\sigma = \frac{(2\pi)^4}{\hbar^2 c^2}\frac{E_0}{k_0} d\vec{k}_n \left|\left\langle t_{K^- n}\right\rangle\right|^2 \times -\frac{1}{\pi}\,\mathrm{Im}\sum\nolimits_n \left\langle \Psi_f^{(n)}\right| \frac{1}{E - H_{K^- p} + i\varepsilon}\left|\Psi_f^{(n)}\right\rangle$$

$$\times \int d\vec{q}_1 \int d\vec{q}_1' \left\langle \Psi_f^{(n)} \middle| \tilde{\vec{q}} \right\rangle \left\langle \tilde{\vec{q}}' \middle| \Psi_f^{(n)} \right\rangle \left\langle \vec{q}_1 \middle| \Psi_i \right\rangle \left\langle \Psi_i^* \middle| \vec{q}_1' \right\rangle. \tag{33}$$

where, $\sum_n \left|\Psi_f^{(n)}\right\rangle\left\langle \Psi_f^{((n))}\right| = 1.$

$$d^3\sigma = \frac{(2\pi)^4}{\hbar^2 c^2}\frac{E_0}{k_0} d\vec{k}_n \left|\left\langle t_{K^- n}\right\rangle\right|^2 \left(-\frac{1}{\pi}\,\mathrm{Im}\int d\vec{q}_1 \int d\vec{q}_1' \left\langle \Psi_i^* \middle| \vec{q}_1' \right\rangle \left\langle \vec{q}_1 \middle| \Psi_i \right\rangle \left\langle \tilde{\vec{q}}' \right| \frac{1}{E - H_{K^- p} + i\varepsilon} \left|\tilde{\vec{q}}\right\rangle\right) \tag{34}$$

where, $\int d\vec{r}\left|\vec{r}\right\rangle\left\langle \vec{r}\right| = 1$.

The equation (34) can be expressed coordinate representation by using completeness relation. It can be expressed as follows

$$\frac{d^2\sigma}{d\cos(\theta)} = \frac{(2\pi)^5}{\hbar^2 c^2}\frac{E_0}{k_0} k_n^2 dk_n \left|\left\langle t_{K^- n}\right\rangle\right|^2 \left(-\frac{1}{\pi}\right)\mathrm{Im}\int d\vec{r}' d\vec{r} f^*(\vec{r}') \left\langle \vec{r}' \right| \frac{1}{E - H_{K^- p} + i\varepsilon} \left|\vec{r}\right\rangle f(\vec{r}) \tag{35}$$

In equation (35) contains dk_n. To get dk_n, the energy conservation law and momentum conservation law are used.

$$dk_n = \frac{Y c^2 dY}{\hbar^2 [(1 + \frac{E_Y}{E_n})k_n - k_0 \cos(\theta)} \tag{36}$$

By substituting equation (36) into equation (35), the equation (35) becomes

$$\frac{d^2\sigma}{dY d\cos(\theta)} = \frac{(2\pi)^5}{\hbar^4}\frac{E_0}{k_0} k_n^2 \left|\left\langle t_{K^- n}\right\rangle\right|^2 \frac{Y}{(1 + \frac{E_Y}{E_n})k_n - k_0 \cos(\theta)} \left(-\frac{1}{\pi}\right)\mathrm{Im}\int d\vec{r}' d\vec{r}$$

$$\times f^*(\vec{r}') \left\langle \vec{r}' \right| \frac{1}{E - H_{K^- p} + i\varepsilon} \left|\vec{r}\right\rangle f(\vec{r}) \tag{37}$$

In equation (37) contains two terms. The first term is kinematical factor and the other one is spectral function.

In this spectral function equation, $\left\langle \vec{r}' \right| \frac{1}{E - H_{K^- p} + i\varepsilon} \left|\vec{r}\right\rangle$ is the Green's function.

The energy conservation term will be solved by using Green's function method.

Numerical Calculation of Green's Function

Green's function can be expressed by coordinate representation as follows;

$$G(\vec{r}', \vec{r}) = \langle \vec{r}' | \frac{1}{E - H_{K^-p} + i\varepsilon} | \vec{r} \rangle . \tag{38}$$

It is satisfies the following equation,

$$\left(E - H_{K^-p}\right) G^+\left(\vec{r}', \vec{r}\right) = \left\langle \vec{r}' | 1 | \vec{r} \right\rangle = \delta\left(\vec{r}' - \vec{r}\right) \tag{39}$$

Green's function and delta function are expressed partial wave expression as

$$G\left(\vec{r}', \vec{r}\right) = \sum_{\ell=0}^{\infty} \sum_M Y_{\ell M}(\hat{\vec{r}}') \frac{G_\ell^{\,+}(r', r)}{r'r} Y^*_{\ell M}(\hat{\vec{r}}) \tag{40}$$

$$\delta\left(\vec{r}' - \vec{r}\right) = \sum_{\ell=0}^{\infty} \sum_M Y_{\ell M}(\hat{\vec{r}}') \frac{\delta(r' - r)}{r'r} Y^*_{\ell M}(\hat{\vec{r}}) \tag{41}$$

Radial part of Green's function $G_\ell^{\,+}(r', r)$ satisfies the following equation,

$$[k^2 + \frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \tilde{U}(r)] G_\ell^{\,+}(r', r) = \frac{2\mu}{\hbar^2} \delta\left(r' - r\right) \tag{42}$$

$k = \sqrt{\frac{2\mu E}{\hbar^2}}$, $\tilde{U}(r) = \frac{2\mu}{\hbar^2} V_{K^-p}(r)$ is the potential of K^-p system.

In our calculation, we have constructed YA (T. Yamazaki and Y. Akaishi (2007))) potential for $\overline{K}N$ interaction.

Integrating equation (44) with $\int dr$ from $r' - \varepsilon$ to $r' + \varepsilon$ will give,

$$\int_{r'-\varepsilon}^{r'+\varepsilon} [k^2 - \frac{\ell(\ell+1)}{r^2} - \tilde{U}(r)] G_\ell^{\,+}\left(r', r\right) dr + \int_{r'-\varepsilon}^{r'+\varepsilon} \frac{d^2}{dr^2} G_\ell^{\,+}\left(r', r\right) dr = \frac{2\mu}{\hbar^2} \int_{r'-\varepsilon}^{r'+\varepsilon} \delta\left(r' - r\right) dr \tag{43}$$

The first term of left hand side vanishes since it is continuous. The second term gives

$$\frac{d}{dr} G_\ell^{\,+}\left(r', r\right)\Big|_{r'+\varepsilon} - \frac{d}{dr} G_\ell^{\,+}\left(r', r\right)\Big|_{r'-\varepsilon} = \frac{2\mu}{\hbar^2} . \tag{44}$$

Green's function $G_\ell^{\,+}\left(r', r\right)$ is divided into two regions $G_{\ell 1}^{\,+}\left(r', r\right)$ and $G_{\ell 2}^{\,+}\left(r', r\right)$ with

$$G_{\ell_1}^{+}\left(r', r\right) = C_1 u_\ell^{(0)}(r) \qquad \left(0 \langle r \langle r'\right)$$

$$G_{\ell_2}^{(+)}\left(r', r\right) = C_1 u_\ell^{(+)}(r) \qquad \left(r' \langle r \langle \infty\right).$$

$u_\ell^{(0)}(r)$ and $u_\ell^{(+)}(r)$ satisfies

$[k^2 + \frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \tilde{U}(r)]u_\ell^{(0)}(r) = 0$ with boundary condition at original $u_\ell^{(0)}(0) \xrightarrow{r\to 0} 0$.

$[k^2 + \frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \tilde{U}(r)]u_\ell^{(+)}(r) = 0$ with boundary condition at asymptotic region,
$u_\ell^{(+)}(r) \xrightarrow{r\to\infty} krh_\ell^{+}(kr)$ (45)

where, $h_\ell^{+}(kr)$ is the spherical Hankel function.

According to the continuity of Green's function,

$$C_1 u_\ell^{(0)}(r)\big|_{r'} = C_2 u_\ell^{(+)}(r)\big|_{r'} . \qquad (46)$$

Discontinuity of $\frac{dG}{dr}$ gives

$$C_1 u_\ell^{(0)'}(r)\big|_{r'} - C_2 u_\ell^{(+)'}(r)\big|_{r'} = \frac{2\mu}{\hbar^2} . \qquad (47)$$

By solving equation (48) and (49), the values of C_1 and C_2 are obtained.

$$C_1 = \frac{\begin{vmatrix} 0 & -u_\ell^{(+)}(r') \\ \frac{2\mu}{\hbar^2} & -u_\ell^{(+)'}(r') \end{vmatrix}}{\begin{vmatrix} u_\ell^{(0)}(r') & -u_\ell^{(+)}(r') \\ u_\ell^{(0)'}(r') & -u_\ell^{(+)'}(r') \end{vmatrix}} = \frac{2\mu}{\hbar^2} \times \frac{u_\ell^{(+)}(r')}{W\left(u_\ell^{(0)}, u_\ell^{(+)}\right)} \qquad (48)$$

$$C_2 = \frac{\begin{vmatrix} u_\ell^{(0)}(r') & 0 \\ u_\ell^{(0)'}(r') & \frac{2\mu}{\hbar^2} \end{vmatrix}}{\begin{vmatrix} u_\ell^{(0)}(r') & -u_\ell^{(+)}(r') \\ u_\ell^{(0)'}(r') & -u_\ell^{(+)'}(r') \end{vmatrix}} = \frac{2\mu}{\hbar^2} \times \frac{u_\ell^{(0)}(r')}{W\left(u_\ell^{(0)}, u_\ell^{(+)}\right)} ,$$

where, $W\left(u_\ell^{(0)}, u_\ell^{(+)}\right)$ is Wronskian. (49).

After comparing the value of r and r ', we used the smaller part is $r_<$ and the larger one is $r_>$.

The final expression of the differential cross section for missing mass spectrum by using Green's function method as shown bellows,

$$\frac{d^2\sigma}{dYd\cos(\theta)} = \frac{(2\pi)^5}{\hbar^4}\frac{E_0}{k_0}k_n^2\left|\left\langle t_{K^-n}\right\rangle\right|^2 \frac{Y}{(1+\frac{E_Y}{E_n})k_n - k_0\cos(\theta)}$$

$$\times\frac{2\mu}{\hbar^2}\left(\frac{-1}{\pi}\right)\mathrm{Im}[\sum_{\ell}(2\ell+1)\int drdr' j_\ell^*(Qr')u_i^*(r')] \qquad (50)$$

YA Potential for $\bar{K}N$ Interaction

The radial form of YA potential for $\bar{K}N$ interaction is as follows:

$V_{\bar{K}N}^{I=0}(r) = (V_0 + i\,W_0)e^{-\left(\frac{r}{b}\right)^2}$, where, V_0 = -597.0 MeV, W_0 = -140.0 MeV, b = 0.66 fm.

It is a Gaussian type complex potential with strength parameters (V_0 and W_0) and range parameter (b), where imaginary part represents decay into $\Sigma\pi$ channel.

Results and Discussion

Missing Mass Spectrum of D (K^-, n) Λ (1405) Reaction

We have calculated the differential cross section for the missing mass spectrum of the $D(K^-,n)$ Λ (1405) reaction (J-PARC E31) (Noumi *et al.*,) by using Green's function method. In our calculation, we used the YA potential for K^-p interaction which reproduced the binding energy and the level width of Λ (1405) quasi-bond state.

By using equation (50), we have analyzed the missing mass spectrum of Λ (1405) for various angular momentum contributions. The calculated missing mass spectrum of $D(K^-,n)\Lambda(1405)$ reaction for individual angular momentum contributions for $\ell = 0$, $\ell = 1$, $\ell = 3$ and $\ell = 8$ are shown in Figure (3). In the energy region below the K^-p threshold, it can be seen that $\bar{K}N$ bound state is mainly contributed by $\ell = 0$. In the continuum region, the higher angular momenta dominantly contribute to the quasi-free peaks. The total angular momentum contributions to the missing mass spectrum converge at $\ell_{max} = 8$ as shown in Figure (4). According to these two figures, it can be concluded that the peak position is found to be clearly dominant at $\ell = 0$. The mass and level width of Λ (1405) is 1407.9 MeV/c^2 and 48 MeV, respectively. The updated PDG value of the mass and width of this Λ (1405) resonance state or often known as Λ* is $1405.1^{+1.3}_{-1.0}$ MeV/c^2 and 50.5 ± 2.0 MeV (Particle Data Group K. A. Olive *et al*., (2014)). The calculated result of the mass and level width of Λ (1405) is nearly consistent with that of the updated PDG (2016) data.

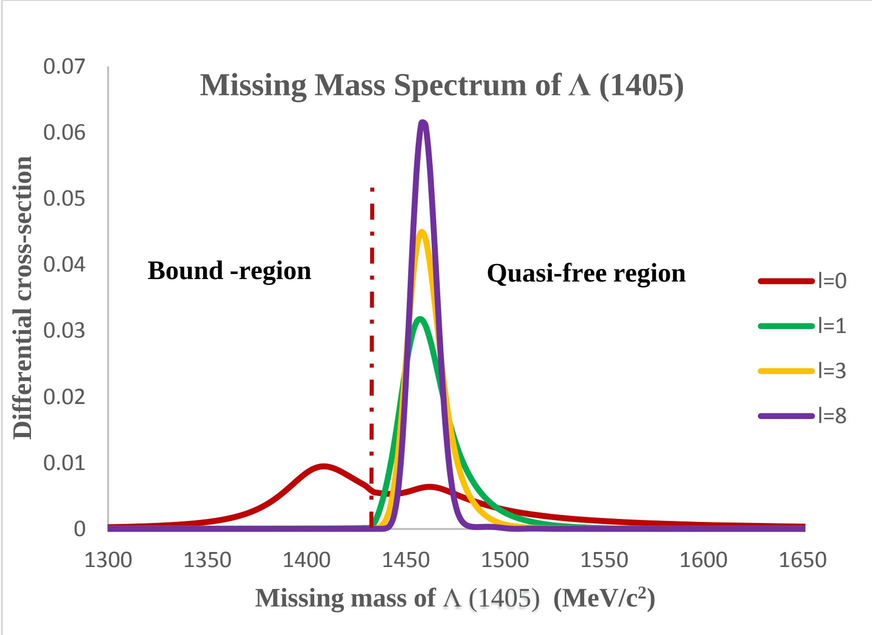

Figure 3 Missing mass spectrum of Λ (1405) with individual angular momentum; the red color solid curve represents $\ell = 0$ only; the green color solid curve represents $\ell = 1$ only; the orange color solid curve represents $\ell = 3$ only; The violet color solid curve represents $\ell = 8$ only

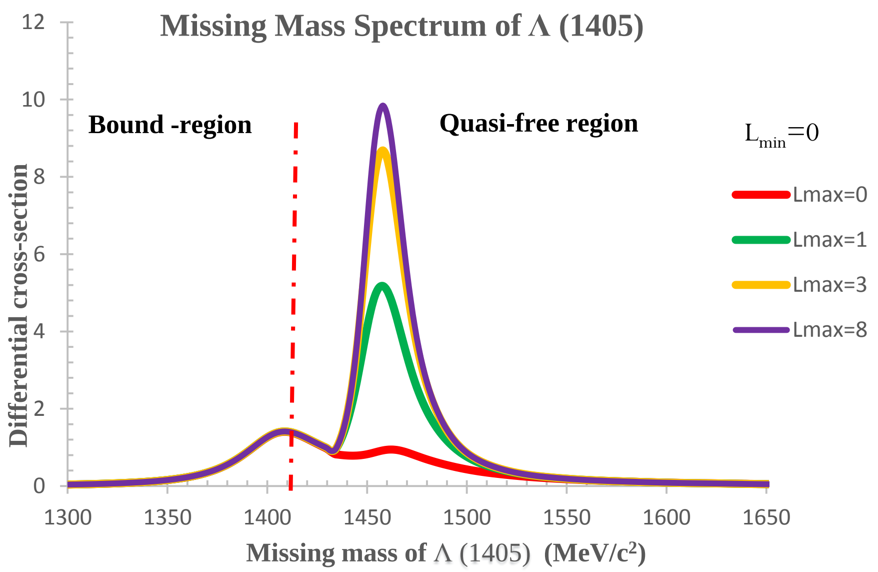

Figure 4 Missing mass spectrum of Λ (1405) with total angular momentum the violet color solid curve represents the summation of total angular momentum $\ell = 0$ to 8; the green color solid curve represents the summation of total angular momentum $\ell = 0$ to 1; the orange color solid curve represents the summation of total angular momentum $\ell = 0$ to 3; the red color solid curve represents the angular momentum $\ell = 0$ only

Conclusion

We have analyzed the missing mass spectrum of D (K^- , n) Λ (1405) reaction at the incident momentum of K^- 1.0GeV/c. The calculated result of the mass and level width of Λ (1405) is nearly consistent with that of the updated PDG (2016) data.

Acknowledgements

I would like to specially thank to my supervisor, Dr Khin Swe Myint, Emeritus Professor, Rector (Rt.d), University of Mandalay, for her invaluable advices and enthusiastic guidance throughout this work.

I do thank Dr Lei Lei Win, Professor and Head of Physics Department, University of Mandalay, for her encouragement and permission.

I also thank my co-supervisor, Dr Thida Wint, Associate Professor, Physics Department, University of Distance Education Mandalay, for her discussions and collaborations.

References

D.W. Thomas *et al*., (1977), Nucl. Phys. **B 129 1**.

H. Noumi *et al*., Spectroscopic study of hyperon resonances below $\bar{K}$ N threshold via the (K^-, n) reaction on deuteron (J-PARC E31), http://j-parc. jp/researcher/Hadron/en/pac_1207/pdf/E31_1202-9.pdf.

J. Esmaili, Y. Akaishi and T. Yamazaki, (2010), Phys. Lett. **B 686 23**.

J. Esmaili, Y. Akaishi and T. Yamazaki, (2011), Phy. Rev. **C 83 055207**.

M.H. Alston et al., (1961), Phys. Rev. Lett. **6 689.**

Maryam Hassanvand, Yoshinori Akaishi, Toshimitsu Yamazaki, (3 Oct 2015), arXiv;**1510.00870v1** [hep-ph].

Particle Data Group K. A. Olive *et al*., (2016), Chin. Phys. **C 40 100001**.

R. H. Dalitz and A. Deloff, (1991), J. Phys. G: Nucl. Part. Phys. **17 289**.

R. J. Hemingway, (1985), Nucl. Phys. **B 253 742**.

T. Hyodo and A. Weise, (2008), Phys. Rev. **C 77 035204**..

T. Yamazaki and Y. Akaishi (2007), Phys. Rev. **C 76 045201.**